**APPENDIX P1**

**3. Introduction of harmonic linearization**

A number of approximation methods have been developed for the analysis of nonlinear systems [4-10]. One such method is the harmonic linearization method. That method is based on the assumption that in the observed system a periodic movement regime is established on the unpaired with unknown amplitude A and oscillation frequency ω. This regime can also occur in systems without feedback but with external periodic excitations.

Self-oscillation mode can be established in non-linear systems with feedback and without the effect of external excitation. This mode is characterized by the fact that all variables in the system are functions of time and as such can be developed into the Fourier series.Any nonlinear system can be represented by a block representing the linearized part L and the nonlinear one as F.The input to the nonlinearity is x and the output is y.

Elementary theory in the field of approximations of nonlinear systems is given in attachment P1 [42].

The output from the nonlinearity can be represented by

(11)

where s≡d/dt is the differentiation operator. For the mentioned configuration, the linear part of the system can have an arbitrary structure.

When determining the self-oscillation parameters A and ω, let the linear part have the property of a low-pass filter. Then for any complex form of the periodic variable y=y(t), it is possible to develop the variable into a Fourier series at the exit from the nonlinearity. That is, represent it as a sum of harmonics, the first of which has a frequency of ω, and the others in order of 2ω, 3ω, etc. Since the linear part is assumed to have a low-pass character, this means that the higher harmonics will be significantly attenuated compared to the fundamental. This allows all higher harmonics to be ignored at the output of the lined section. Based on that fact, it is assumed that the variable x=x(t) represents simple periodic oscillation of some frequency ω and amplitude A and that

(12)

The output from the nonlinearity is now a complex periodic function

(13)

where C0,A1,B1,A2,B2,.. . are the Fourier coefficients. Let's assume that the mean value of the variable y is equal to zero, that is, we have a relation of the form

(14)

This case will always be valid when the characteristic of the nonlinear element is symmetric with respect to the coordinate origin and when the system is not affected by an external disturbance. Since the higher harmonics are neglected, the relation can be written

(15)

Based on the previous relations, they are derived

(16)

on the basis of which the output from the nonlinearity of the shape is obtained

(17)

The coefficients

are calculated from the relations

(18)

The coefficients from (18) were obtained from the already known expressions for the coefficients of the Fourier series A1 and B1. In this way, we approximated the nonlinear characteristic to a linear one according to the relation (17). The coefficients from (18) are functions of the self-oscillation parameters A and ω. That is why the coefficients are called harmonic linearization coefficients.

Expression of form

(11)

it is called the descriptive function of the nonlinearity. These coefficients can be calculated for each of the nonlinearities that are characteristic.

From picture no. 2, starting from the differential equations that describe the dynamic behavior of the linear part of the system, it is possible to form the equation

(12)

where Q(s) and P(s) are polynomials by the differentiation operator s. With zero initial conditions, the transfer function of the linear part of the system given as P(s)/ Q(s) can be formed. We call such a function the equivalent linear part. If we proceed from (12) with relation (11), it is possible to form a general differential equation of the form

(13)

where s≡d/dt is the differentiation operator.

For the mode of self-oscillations, the amplitude A and the frequency ω are constant values. Then for periodic solutions, with x≠0, they are obtained from the roots of the characteristic equation

(14)

To find the mentioned parameters, the Mihailov curve can be used, so that the polynomial (14) is used, where for s=jω, the characteristic equation is obtained

, (15)

where ω\* is a current parameter and which should not be equated with the required frequency of self-oscillations ω.By separating the real and imaginary parts of equation (15), we get

(16)

In order for (14) to have purely imaginary solutions, the Mihailov curve must pass through the coordinate origin, where the frequency of self-oscillations ω is determined for the value ω\*in the coordinate origin. This means that a condition must be met

(17)

by means of which the parameters of self-oscillations are found. In the case that A and ω are positive, this means that a mode of self-oscillations has arisen. Analytically, it is often difficult to find solutions, so one resorts to finding values graphically. In this case, since it is a deliberately chosen complex nonlinear system, numerical methods with computer programming will be used to solve it.